THE COMPLETE **HIGH SCHOOL** STUDY GUIDE

EVERYTHING YOU NEED TO ACE PRE-ALGEBRA & ALGEBRA 1 IN ONE BIG FAT NOTEBOOK

Like notes borrowed from the **SMARTEST KID** in **CLASS**
(Double-checked by an **AWARD-WINNING** teacher)

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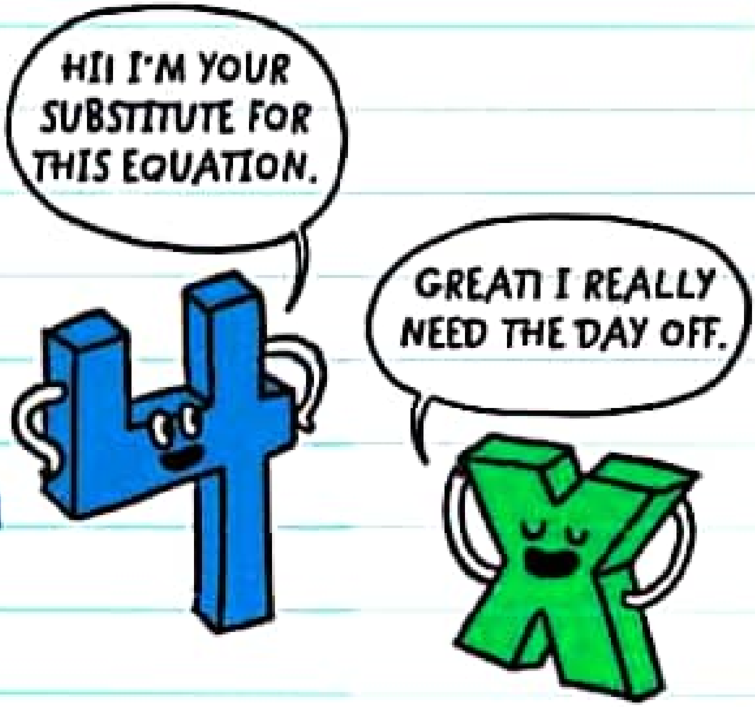

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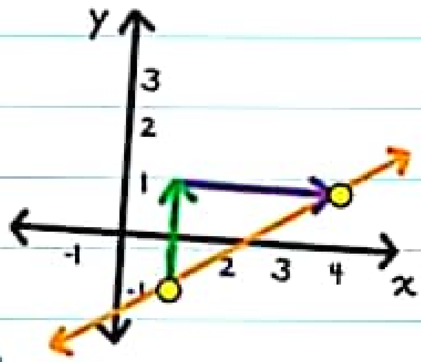

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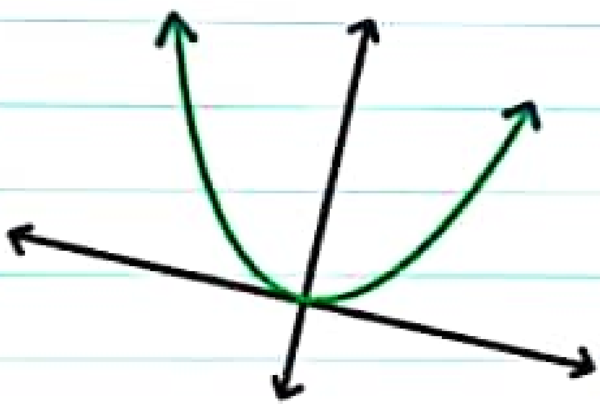

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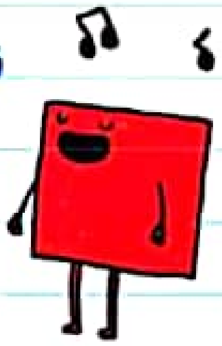

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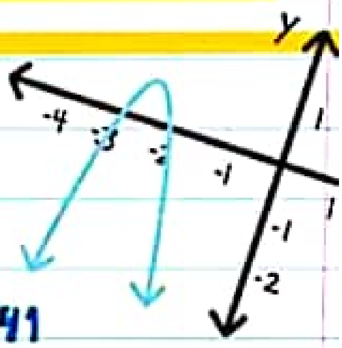

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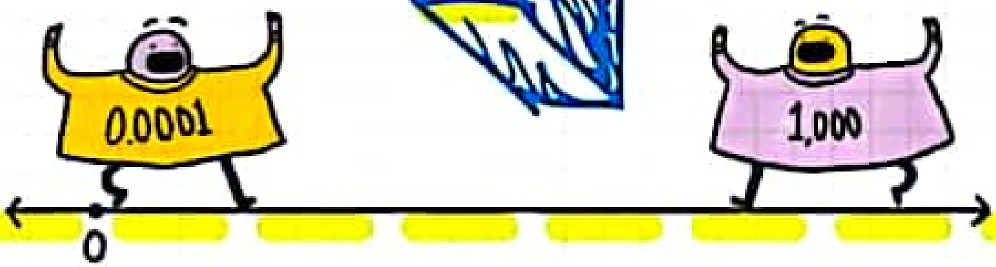
Unit 1



Arithmetic Properties

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Chapter 1

TYPES OF NUMBERS

All numbers can be classified into various categories. Here are the categories that are most often used in mathematics:

NATURAL NUMBERS or Counting Numbers: The set of all positive numbers starting at 1 that have no fractional or decimal part; also called whole numbers.

Examples: 1, 2, 3, 4, 5, . . .

WHOLE NUMBERS: The set of all natural numbers and 0.

Examples: 0, 1, 2, 3, 4, 5, . . .

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INTEGERS: The set of all whole numbers, including negative natural numbers.

Examples: . . . -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, . . .

RATIONAL NUMBERS: The set of all numbers that can be written by dividing one integer by another. These include any number that can be written as a fraction or ratio.

Note: You cannot have 0 in the denominator of a fraction.

Examples: $\frac{1}{2}$ (which equals $\frac{1}{2}$ or $\frac{-1}{-2}$)

$0.\overline{3}$ (which equals $\frac{1}{3}$)

This means that the number below repeats forever.

-8 (which equals $\frac{-8}{1}$ or $\frac{8}{-1}$)

3.27 (which equals $\frac{327}{100}$)

Natural numbers, whole numbers, and integers are all rational numbers.

IRRATIONAL NUMBERS: The set of all numbers that are *not* rational numbers. These are numbers that cannot be written by dividing one integer by another. When we write an irrational number as a decimal, it goes on forever, without repeating itself.

"..." means that the number continues on forever.

Examples: $\sqrt{5} = 2.2360679774997\ldots$ $\pi = 3.141592653\ldots$

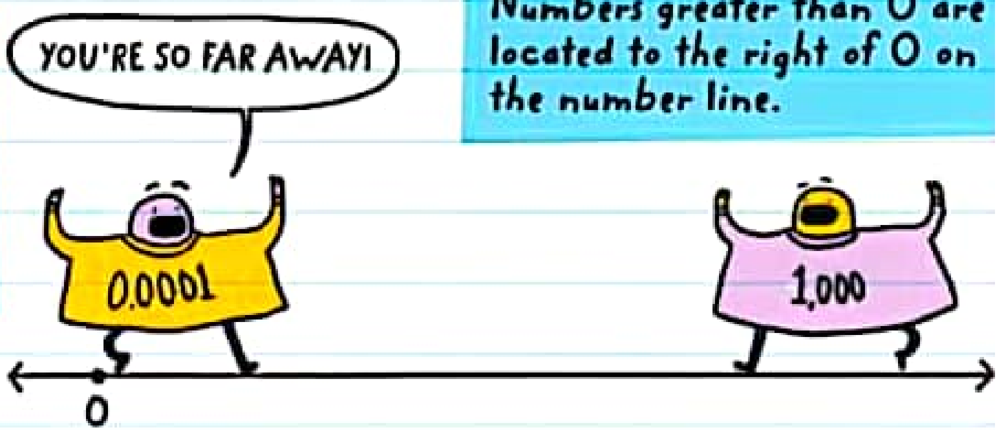
0.25 is NOT irrational because it terminates or ends.

0.347157157157I5 . . . is NOT irrational because the digits repeat themselves.

REAL NUMBERS: The set of all numbers on a number line. Real numbers include all rational and irrational numbers. This can be zero, positive or negative integers, decimals, fractions, etc.

Examples: $8, -19, 0, \frac{3}{2}, \sqrt{47}, \sqrt{25}, \pi, \ldots$

Numbers less than zero are located to the left of 0 on the number line.



Numbers greater than 0 are located to the right of 0 on the number line.

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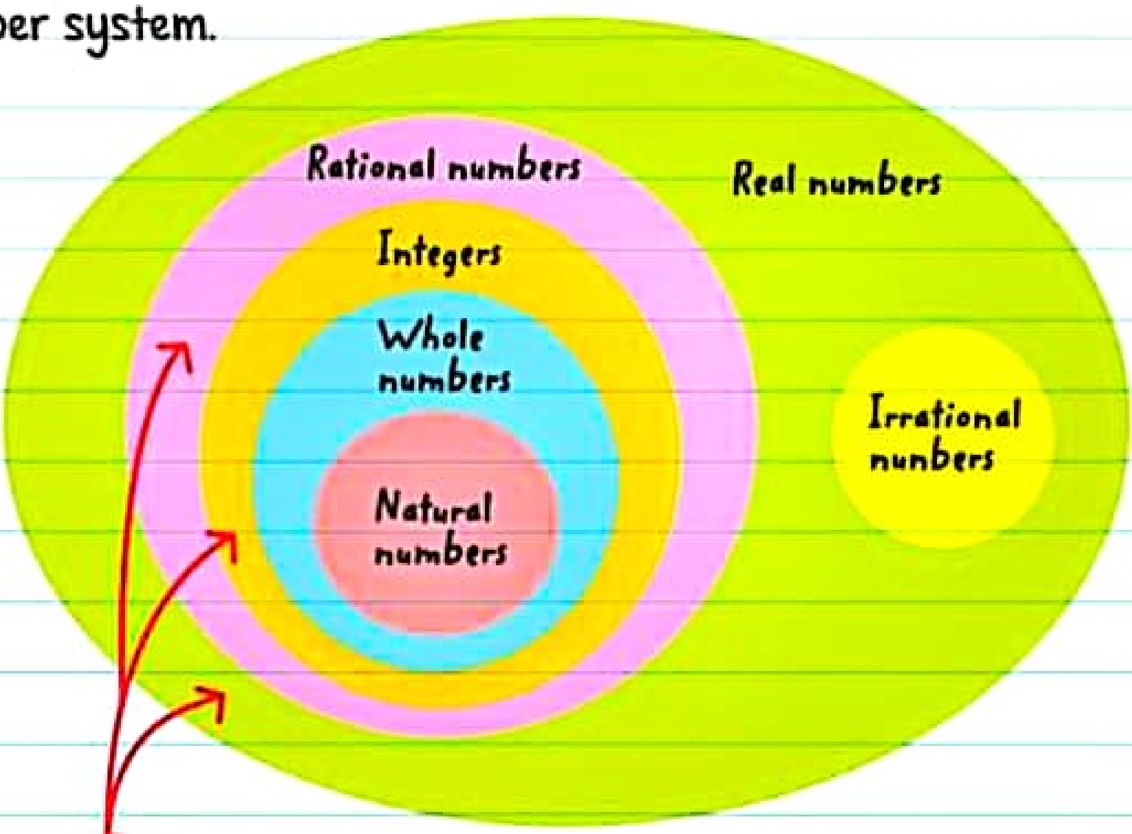
Here's how all the types of numbers fit together in our number system.



Example: −2 is an integer, a rational number, and a real number

Some other examples:

24 is natural, whole, an integer, rational, and real.

0 is whole, an integer, rational, and real.

$\frac{2}{3}$ is rational and real.

6.675 is rational and real.

$\sqrt{5} = 2.2360679774997\ldots$ is irrational and real.

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SOME IMPORTANT POINTS ABOUT DECIMALS

1. **Terminating decimals** are decimals that have no repeating digit or group of digits.

All terminating decimals are rational numbers.

To **terminate** means to end.

Example: 0.25

the decimal ends

2. **Repeating decimals** are decimals that go on infinitely, but one or more digits repeat themselves. All repeating decimals are rational numbers.

Examples: $\frac{1}{3} = 0.\overline{3}$ or $\frac{9}{7} = 1.\overline{285714}$

$\frac{9}{7} = 1.285714285714\ldots$

The bar over the digits "285714" means that all of those digits repeat infinitely.

3.14159265535...

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CHECK YOUR KNOWLEDGE

For questions 1 through 10, classify each number in as many categories as possible.

1. 62

2. $\frac{8}{10}$

3. 9.28519692714385 . . .

4. 0

5. 3.7

6. -260

7. $-\frac{5}{2}$

8. π

9. $3.25\overline{197}$

10. $\sqrt{49}$

ANSWERS

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CHECK YOUR ANSWERS

1. natural, whole, integer, rational number, real number

2. rational, real

3. irrational, real

4. whole, integer, rational, real

5. rational, real

6. integer, rational, real

7. rational, real

8. irrational, real

9. rational, real

10. Since $\sqrt{49}$ is equal to 7, it is a natural number, whole number, integer, rational number, and real number.

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Chapter 2

ALGEBRAIC PROPERTIES

BASIC PROPERTIES

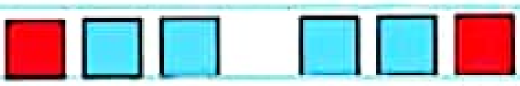
The **Commutative Property of Addition** and the **Commutative PROPERTY OF MULTIPLICATION** tell us that when we are adding two numbers or multiplying two numbers, the order of the numbers does not matter to get a correct calculation.

Think: To commute means to move around. So we can move the order of numbers around and not affect the result.

The **COMMUTATIVE PROPERTY OF ADDITION** states that for any two numbers a and b: $a + b = b + a$.

These are equivalent numerical expressions. This means that both sides of the math equation have equal value.

Example: $1 + 2 = 2 + 1$



$$3\frac{2}{7} + 1\frac{5}{6} = 1\frac{5}{6} + 3\frac{2}{7}$$

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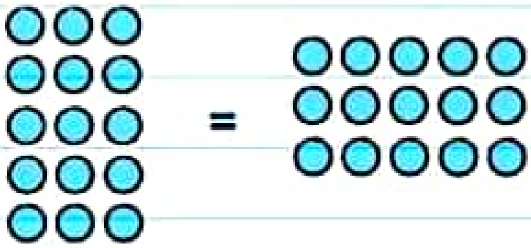
The **COMMUTATIVE PROPERTY OF MULTIPLICATION** states that for any two numbers x and y: $x \cdot y = y \cdot x$.

Example: $5 \cdot 3 = 3 \cdot 5$



> The Commutative Properties work only with addition and multiplication; they do not work with subtraction and division.

The **Associative Property of Addition** and the **Associative Property of Multiplication** tell us that when we are adding three numbers or multiplying three numbers, the order in which we group the numbers does not matter.

The **ASSOCIATIVE PROPERTY OF ADDITION** states that for any three numbers a, b, and c: $(a + b) + c = a + (b + c)$.

For example, $1 + 2 + 5$ can be calculated either as:

$(1 + 2) + 5 = 3 + 5 = 8$

or

The grouping doesn't matter.
The sum is the same.

$1 + (2 + 5) = 1 + 7 = 8$

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The **ASSOCIATIVE PROPERTY OF MULTIPLICATION** states that for any three numbers a, b, and c:

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

For example, $2 \cdot 3 \cdot 5$ can be calculated either as:

$(2 \cdot 3) \cdot 5 = 6 \cdot 5 = 30$

or

$2 \cdot (3 \cdot 5) = 2 \cdot 15 = 30$

Grouping doesn't matter. The product is the same.

The Associative Properties work only with addition and multiplication; they do not work with subtraction and division.

What's the difference between commutative properties and associative properties?

Commutative relates to the order of the numbers.

Associative relates to the grouping of the numbers.

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The **DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION** says that we get the same number when we multiply a group of numbers added together or when we multiply each number separately and then add them.

> **Think:** Distributive means to share or give out.

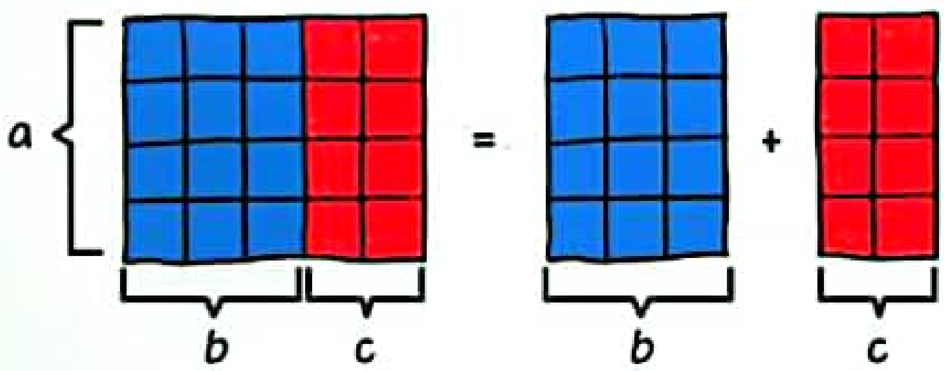
- The Distributive Property can be used when multiplying a number by the sum of two numbers:

Given three numbers a, b, and c: $a(b + c) = (a \cdot b) + (a \cdot c)$.

$a(b + c) = a \cdot b + a \cdot c$ We are **DISTRIBUTING** the term a to each of the terms b and c.

The *Distributive Property* states:

Adding two numbers inside the parentheses and then multiplying that sum by a number outside the parentheses *is the same as* first multiplying the number outside the parentheses by each of the addends inside the parentheses and then adding the two products together.



EXAMPLE: Use the Distributive Property to expand and then simplify $3(6 + 8)$.

$3(6 + 8) = 3 \cdot 6 + 3 \cdot 8$ expand

$= 18 + 24 = 42$ simplify

The **DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER SUBTRACTION** says that we get the same number when we multiply a group of numbers subtracted together or when we multiply each number separately and subtract them.

Given three numbers a, b, and c: $a(b - c) = (a \cdot b) - (a \cdot c)$.

So, $a(b - c) = a \cdot b - a \cdot c$

EXAMPLE: Use the Distributive Property to expand and then simplify $2(10 - 7)$.

$2(10 - 7) = 2 \cdot 10 - 2 \cdot 7 = 20 - 14 = 6$

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- The Distributive Property can also be used for expressions with multiple terms.

To expand $a(b + c - d)$:

$$a(b + c - d) = a \cdot b + a \cdot c - a \cdot d = ab + ac - ad$$

EXAMPLE: Use the Distributive Property to expand and simplify $6(2 - 1 + 5)$.

$$6(2 - 1 + 5) = 6(2) - 6(1) + 6(5)$$
$$= 12 - 6 + 30 = 36$$

Note: The Distributive Property does **NOT** work for division!

Examples:

$$a \div (b + c) \neq a \div b + a \div c$$

$$40 \div (8 + 2) \neq 40 \div 8 + 40 \div 2$$

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CHECK YOUR KNOWLEDGE

For questions 1 through 4, state the property used.

1. $3 \cdot 5 = 5 \cdot 3$

2. $(a + b) + \frac{1}{2} = a + (b + \frac{1}{2})$

3. $\frac{1}{2} \cdot (5 \cdot \frac{4}{3}) = (\frac{1}{2} \cdot 5) \cdot \frac{4}{3}$

4. $0 + 5 = 5 + 0$

For problems 5 through 6, state whether or not the property is being applied correctly.

5. Use the Associative Property to state: $\frac{1}{2} \div 7 = 7 \div \frac{1}{2}$

6. Use the Associative Property to state: $7 + 3 - 1$ can be calculated either as:
$(10 - 3) - 1$ or $10 - (3 - 1)$

For questions 7 through 10, use the Distributive Property to expand each expression, then simplify your answer.

7. $2(3 + 8)$

8. $m(n) - m(12) = mn - 12m$

9. $4(10 - 2 + 5)$

10. $x(y) - x(2) + x(3) =$
$xy - x2 + 3x$

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CHECK YOUR ANSWERS

1. Commutative Property of Multiplication
2. Associative Property of Addition
3. Associative Property of Multiplication
4. Commutative Property of Addition
5. Not correct. The Associative and Commutative Properties cannot be used for division.
6. Not correct. The expression results in different answers.
7. $2(3) + 2(8) = 6 + 16 = 22$
8. $mn - 12m$
9. $4(10) - 4(2) + 4(5) = 40 - 8 + 20 = 52$
10. $xy - xz + 3x$

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Chapter 3

ORDER OF OPERATIONS

The order of operations is an order agreed upon by mathematicians. It directs us to perform mathematical calculation in the following order:

1ST Any calculations inside parentheses or brackets

2ND Exponents, roots, and absolute value are calculated left to right

3RD Multiplication and division—whichever comes first when you calculate left to right

4TH Addition and subtraction—whichever comes first when you calculate left to right

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You can use the mnemonic "**Please Excuse My Dear Aunt Sally**" for the acronym **PEMDAS** (Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction) to remember the order of operations, but it can be VERY misleading.

This is because you can do division before multiplication or subtraction before addition, as long as you are calculating **from left to right**.

Also, because other calculations like roots and absolute value aren't included, PEMDAS isn't totally foolproof.

EXAMPLE: Simplify the expression: $7 - 4 + 1$

$= \boxed{7 - 4} + 1$

$= 3 + 1$

$= 4$

First, do subtraction or addition, whatever comes first, left to right.

PARENTHESES

EXPONENTS

MULTIPLICATION (left to right)

DIVISION (left to right)

ADDITION (left to right)

SUBTRACTION (left to right)

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EXAMPLE: Simplify the expression: $10 - 3 \times 2$

$= 10 - \boxed{3 \times 2}$

$= 10 - 6$

$= 4$

First, do multiplication.
(PEMDAS: multiplication before subtraction)

EXAMPLE: Simplify the expression: $(9 + 3) \div 1.5$

$= \boxed{(9 + 3)} \div 1.5$

$= 12 \div 1.5$ or $\frac{12}{1.5}$

$= 8$

First, do the operation inside the parentheses.

EXAMPLE: Simplify the expression: $84 - 72 \div 6 \times 2 + 1$

$= 84 - \boxed{72 \div 6} \times 2 + 1$

$= 84 - \boxed{12 \times 2} + 1$

$= \boxed{84 - 24} + 1$

$= 60 + 1$

$= 61$

Note: Another way to think of this problem is by using a fraction bar: $\frac{72}{6}$

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EXAMPLE: Alice's basketball team makes 8 regular two-point shots and 4 three-point shots. Bob's basketball team makes 10 two-point and 2 three-point shots. How many more total points did Alice's team score than Bob's team?

Calculate the total points Alice's team made:

$[(8 \cdot 2) + (4 \cdot 3)]$

Calculate the total points Bob's team made:

$[(10 \cdot 2) + (2 \cdot 3)]$

Subtract the two scores:

$= [(8 \cdot 2) + (4 \cdot 3)] - [(10 \cdot 2) + (2 \cdot 3)]$
$= (16 + 12) - [(10 \cdot 2) + (2 \cdot 3)]$
$= 28 - [(10 \cdot 2) + (2 \cdot 3)]$
$= 28 - (20 + 6)$
$= 28 - 26$
$= 2$

Alice's team scored 2 more points than Bob's team.

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CHECK YOUR KNOWLEDGE

For problems 1 through 8, simplify each expression.

1. $9 - 12 \div 3$

2. $21 - 5 \times 3 + 7$

3. $5 \times (13 - 7) \div 2$

4. $64 - 16 \div 2$

5. $1.8 \div 0.03 - (0.5)(0.4)$

6. $8 \div 16 \times 0.28 - (0.2)(0.2)$

7. $\frac{7}{8} - \frac{1}{2} \times \frac{5}{6}$

8. $\frac{1}{12} - \frac{7}{6} \div \frac{5}{8} \times 1\frac{1}{2}$

9. Carl buys 3 pens, 4 notebooks, and 7 binders. Daria buys 9 pens, 6 notebooks, and 5 binders. Pens cost \$2 each, notebooks cost \$1.50 each, and binders cost \$2 each. How much do Carl and Daria spend altogether?

10. We always multiply before we divide. True or False?

ANSWERS

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CHECK YOUR KNOWLEDGE

1. 5
2. 13
3. 15
4. 56
5. 59.8
6. 0.1
7. $\frac{11}{24}$
8. $-2\frac{43}{60}$
9. $63
10. False. We choose whether to multiply or divide first based on which comes first, left to right.

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Unit 2

The Number System

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Chapter 4

ADDING POSITIVE AND NEGATIVE WHOLE NUMBERS

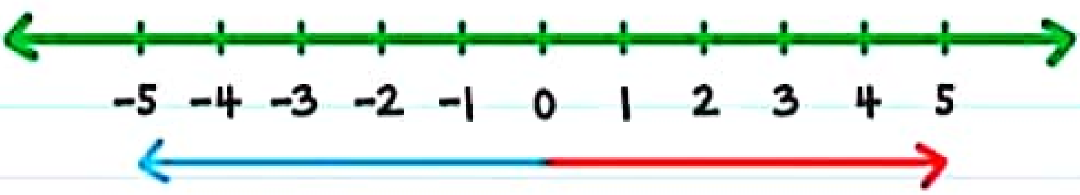
POSITIVE NUMBERS describe quantities greater than zero. Positive numbers are shown with and without the positive sign. For example, +2 and 2.

NEGATIVE NUMBERS describe quantities less than zero. All negative numbers have a negative sign in front of them. For example, -6.

There are various ways to add positive and negative numbers.

METHOD #1:

USE A NUMBER LINE



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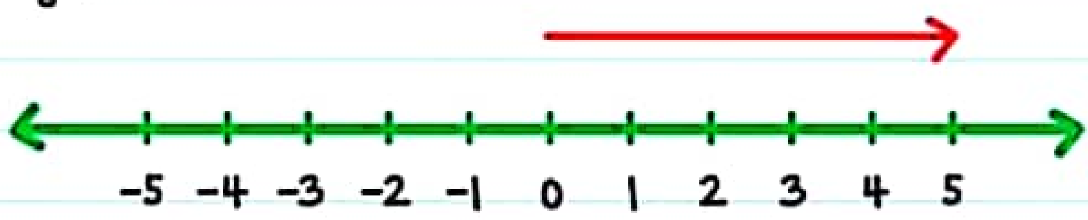
Draw a number line. Begin at zero.

For a POSITIVE (+) number, x, move x units to the right.
For a NEGATIVE (-) number, $-y$, move y units to the left.

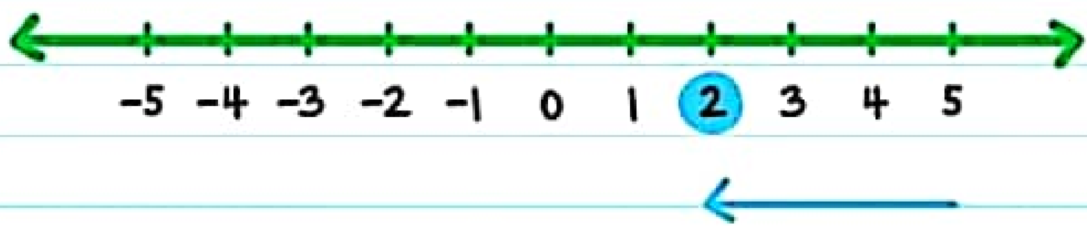
Whichever position you end up at is the answer.

EXAMPLE: Find the sum: 5 + (-3).

1. Begin at zero. Since 5 is a positive number, move 5 units to the right.



2. Begin where you left off with the first number. Since -3 is a negative number, start at 5 and move 3 units to the left.



We end up at 2.

The sum of 5 and -3 is 2.

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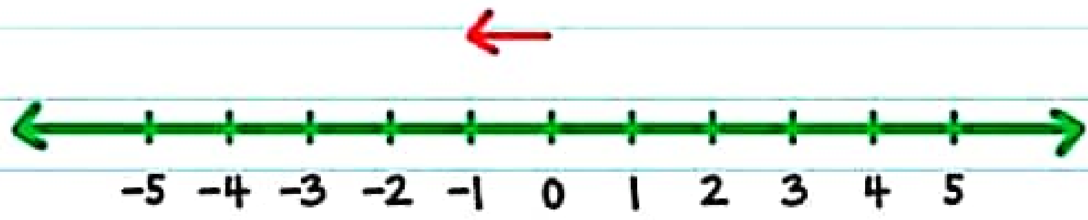
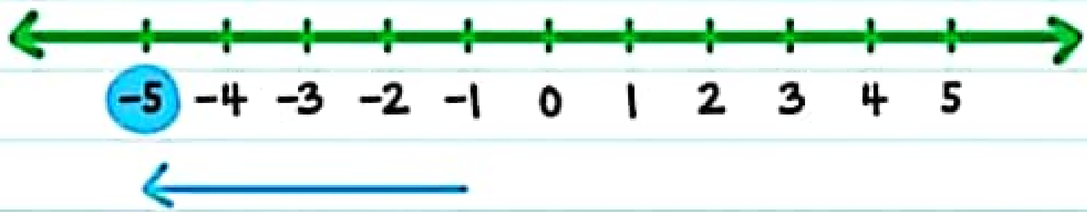
EXAMPLE: Find the sum: (−1) + (−4).

1. Begin at zero. Since −1 is a negative number, move 1 unit to the left.



2. Because −4 is a negative number, move 4 units to the left starting at −1.

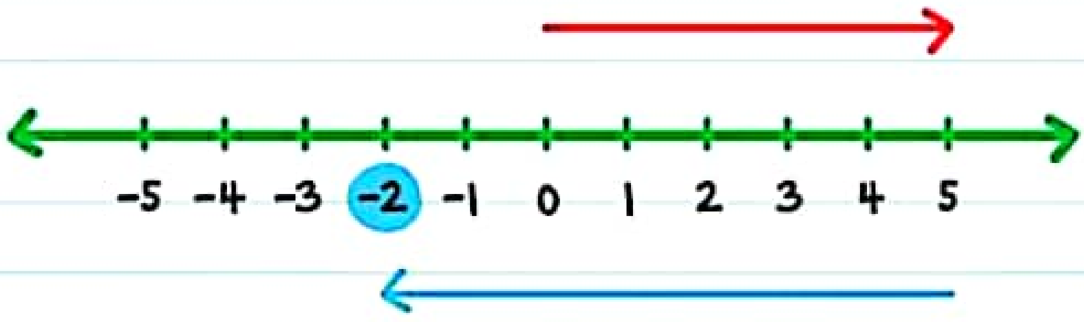


We end up at −5.

The sum of −1 and −4 is −5.

EXAMPLE: Find the sum: $5 + (-7)$.

Move 5 units to the right. Then move 7 units to the left.



We end up at -2.

The sum of 5 and -7 is -2.

The sum of a number and its opposite always equals zero.
For example, $8 + (-8) = 0$.

METHOD #2:

USE ABSOLUTE VALUE

The absolute value of a number represents the distance of that number from zero on the number line.
It's always positive because distance is always positive!

If the signs of the addends are the same, it means that they move in the same direction on the number line. This means that you can add those two numbers together and keep the sign that they share.

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EXAMPLE: Find the sum: (-1) + (-4).

Both -1 and -4 are negative, so they are alike.

We can add them together and keep their sign to get: -5.

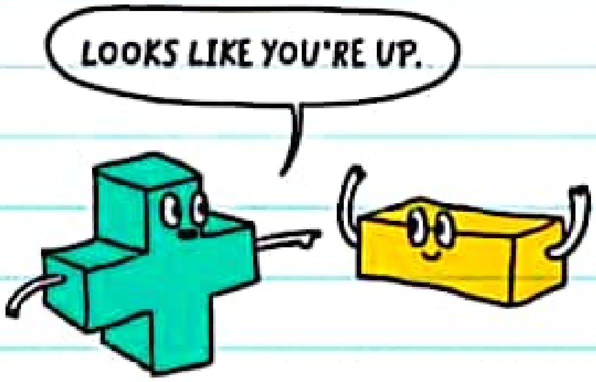
If the signs of the addends are different, it means that they move in opposite directions on the number line. This means you can subtract the absolute value of each of the two numbers. The answer will have the same sign as the number with the greater absolute value.

EXAMPLE: Find the sum: (-11) + 5.

-11 and 5 have different signs, so subtract the absolute value of -11 and the absolute value of 5:

$$|-11| - |5| = 11 - 5 = 6$$

-11 has the greater absolute value, so the answer is also negative: -6.



EXAMPLE: An archaeologist is studying ancient ruins. She brings a ladder to study some artifacts found above ground level and some found below ground level. The archaeologist first climbs the ladder to 5 feet above ground level to study artifacts found in a wall. She then climbs the ladder another 2 feet higher. Finally, the archaeologist climbs down the ladder 11 feet. Where does the archaeologist end up?

First, assign integers to the archaeologist's movements.

Climbs 5 feet above ground level: +5

Climbs another 2 feet above: +2

Climbs down 11 feet: −11

Write an equation to show the archaeologist's movements.

$= 5 + 2 + |-11|$
$= 7 + |-11|$
$= -4$

The archaeologist ends up 4 feet below ground level.

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CHECK YOUR KNOWLEDGE

For problems 1 through 7, find the sum of each expression.

1. $8 + (-3)$
2. $-7 + 3$
3. $-6 + (-8)$
4. $-7 + 9$
5. $-10 + (-9)$
6. $(-5) + (-8)$
7. $9 + (-14)$
8. A hiker is currently in a valley that is at an elevation of 50 feet below sea level. She hikes up a hill and increases her elevation 300 feet. What is the new elevation of the hiker?

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9. A submarine pilot is currently at a depth of 75 feet below sea level. He then pilots his submarine 350 feet lower. What is the new depth of the pilot?

For problem 10, state whether the statement is true or false.

10. Kris is asked to find the sum of (−8) + 5. Kris says:
"Since the numbers have opposite signs, we subtract the absolute value of the numbers: $|-8| - |5| = 8 - 5 = 3$*. Therefore, the answer is: 3."*

ANSWERS

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CHECK YOUR ANSWERS

1. 5
2. -4
3. -14
4. 2
5. -19
6. -13
7. -5
8. 250 feet above sea level
9. 425 feet below sea level
10. False. Since -8 has the larger absolute value, the answer is negative.

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Chapter 5

SUBTRACTING POSITIVE AND NEGATIVE WHOLE NUMBERS

Subtraction and addition are inverse operations. To solve a subtraction problem we can change it to an addition problem by using the **ADDITIVE INVERSE**.

ADDITIVE INVERSE
the number you add to a given number to get zero

EXAMPLE: Find the difference: 7 − 3.

= 7 − 3
= 7 + (−3)
= 4

Change the subtraction problem into an addition problem. Add the additive inverse.

EXAMPLE: Find the difference: $-3 - 5$.

$= -3 - 5$

$= -3 + (-5)$

$= -8$

Change the subtraction problem into an addition problem.
-5 is the additive inverse of 5.

EXAMPLE: Find the difference: $-7 - (-6)$.

$= -7 - (-6)$

$= -7 + (6)$

$= -1$

Change the subtraction problem into an addition problem.
6 is the additive inverse of -6.

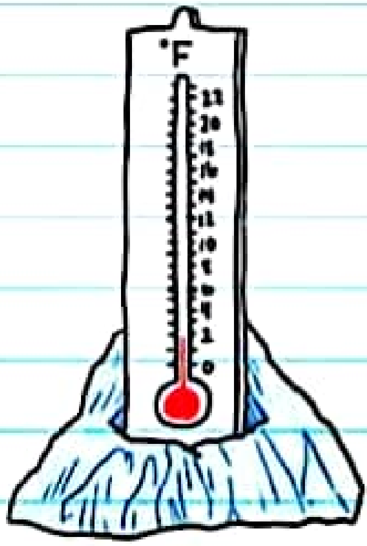
EXAMPLE: The temperature in North Dakota was 5°F in the afternoon. By night, the temperature had decreased by 12 degrees. What was the temperature at night?

Since the temperature decreased, we use subtraction to find the answer:

$= 5 - 12$

$= 5 + (-12)$

$= -7$



The temperature at night was −7°F.

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CHECK YOUR KNOWLEDGE

For problems 1 through 8, find the difference for each expression.

1. $3 - 9$
2. $5 - 7$
3. $-2 - 5$
4. $-10 - 4$
5. $8 - (-5)$
6. $-7 - (10)$
7. $9 - (-20)$
8. $(-12) - (-15)$

For 9 through 10, answer each problem using the subtraction of integers.

9. Sam guesses that his store's average profit is \$17 per hour. However, his store's actual average profit is –\$6 per hour. How far apart is the error in his analysis?

10. A window washer is 110 feet above sea level. A diver is 70 feet below sea level. How many feet apart are the window washer and the diver?

ANSWERS

CS CamScanner

CHECK YOUR ANSWERS

1. -6

2. -2

3. -7

4. -14

5. 13

6. -17

7. 29

8. 3

9. The error is $23 apart.

10. The window washer and the diver are 180 feet apart.

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Chapter 6

MULTIPLYING AND DIVIDING POSITIVE AND NEGATIVE WHOLE NUMBERS

When multiplying or dividing positive and negative numbers: First, count the number of negative signs. Then multiply or divide the numbers.

If there is an ODD NUMBER of negative signs, then the answer is NEGATIVE.

If there is an EVEN NUMBER of negative signs, then the answer is POSITIVE.

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There is an odd number (1) of negative signs, so the answer is negative. — $(+) \times (-) = (-)$

$(-) \div (+) = (-)$ — There is an odd number (1) of negative signs, so the answer is negative.

There are an even number (2) of negative signs, so the answer is positive. — $(-) \times (-) = (+)$

EXAMPLES:

Calculate the product of $4 \times (-5)$.

$= 4 \times (-5)$

$= -(4 \times 5)$ There is 1 negative sign. So, the answer is negative.

$= -20$

Calculate the quotient of $(-91) \div (-7)$.

$= (-91) \div (-7)$ There are 2 negative signs. So, the answer is positive.

$= (91 \div 7)$ Divide 91 by 7.

$= 13$

Ray's credit card balance decreases by \$14 each month. How much will his balance decrease by after 9 months?

$9 \times (-14)$

$-(9 \times 14) = -126$

Ray's credit card balance will have decreased by \$126 after nine months.

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There are 2 negative signs, so the answer is positive. — $(+) \div (-) \div (-) = (+)$

$(-) \times (+) \div (-) \times (-) \times (-) \div (+) \div (-) = (-)$

There are 5 negative signs, so the answer is negative.

The same rule applies when multiplication and division are in the same expression.

EXAMPLE:

Simplify $20 \div (-5) \times (-2)$.

$= 20 \div (-5) \times (-2)$ — There are 2 negative signs. So, the answer is positive.

$= (20 \div 5 \times 2)$ — Multiply or divide—whatever comes first—left to right. So divide!

$= (4 \times 2)$

$= 8$

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CHECK YOUR KNOWLEDGE

For questions 1 through 8, simplify each expression.

1. $7 \times (-12)$

2. $(-84) \div (-12)$

3. $2 \times (-1) \times (-7)$

4. $(-5)(-2)(-3)(0)(-8)$

5. $(-42) \div (-3)$

6. $(-84) \div (-7) \div (-3)$

7. $(-80) \div (-5) \div (-2) \div (-1) \div (-4)$

8. $(-32) \div (-8) \div (-2)$

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For questions 9 and 10, answer each problem using the multiplication or division of integers.

9. Mary drops a penny into a pond. The penny drops 1.5 inches every second. How many inches below the surface will it be after 8 seconds?

10. Patricia randomly picks a negative number. She then decides to multiply that negative number by itself over and over, for a total of 327 times. What sign will the final answer have?

ANSWERS

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CHECK YOUR ANSWERS

1. -84

2. 7

3. 14

4. 0

5. 14

6. −4

7. -2

8. -2

9. The penny will be 12 inches below the surface.

10. The answer will be negative.

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Chapter 7

MULTIPLYING AND DIVIDING POSITIVE AND NEGATIVE FRACTIONS

Multiplying and dividing positive and negative fractions uses the same method that we used with whole numbers:

1. First, count the number of negative signs to determine the sign of the product or quotient.

2. Convert any mixed numbers into improper fractions.

3. Last, multiply or divide the fractions without the negative sign.

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When multiplying fractions, you sometimes might see that one fraction's numerator and another fraction's denominator have common factors.

You can simplify those numbers in the same way that fractions are simplified, by dividing both numbers by the Greatest Common Factor (GCF).

This is called **CROSS-REDUCING** or **CROSS-CANCELING**.

EXAMPLE: Find the product:

$$\left(-2\frac{2}{5}\right) \times \left(-3\frac{1}{3}\right) \times \left(-\frac{8}{9}\right)$$

There are 3 negative signs, so the answer is negative.

$$= -\left(2\frac{2}{5} \times 3\frac{1}{3} \times \frac{8}{9}\right)$$

Convert the mixed numbers to improper fractions.

$$= -\left(\frac{12}{5} \times \frac{10}{3} \times \frac{8}{9}\right)$$

$$= -\left(\frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{3}}} \times \frac{8}{9}\right)$$

The GCF of 12 and 3 is 3:
$12 \div 3 = 4$ and $3 \div 3 = 1$

The GCF of 10 and 5 is 5:
$10 \div 5 = 2$ and $5 \div 5 = 1$

$$= -\frac{64}{9} = -7\frac{1}{9}$$

Rewrite improper fraction as a mixed number.

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EXAMPLE: Zoe needs $3\frac{1}{5}$ feet of fabric to make a tall hat. If Zoe wants enough fabric to make $2\frac{1}{2}$ tall hats, how much fabric will Zoe need?

$= 3\frac{1}{5} \times 2\frac{1}{2}$

Change the mixed numbers to improper fractions.

$= \frac{16}{5} \times \frac{5}{2}$

$= \frac{\overset{8}{\cancel{16}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}}$

The GCF of 16 and 2 is 2:
$16 \div 2 = 8$ and $2 \div 2 = 1$

$= 8$

Zoe will need 8 feet of fabric.

DIVIDING POSITIVE AND NEGATIVE FRACTIONS

When dividing fractions, rewrite the division problem as a multiplication problem by finding the reciprocal of the second number.

When a number is multiplied by its **RECIPROCAL**, the resulting product is 1. For example, the reciprocal of 8 is $\frac{1}{8}$. If you multiply the two numbers, you get 1.

$$\frac{8}{1} \times \frac{1}{8} = 1$$

EXAMPLE: Calculate the quotient of $\frac{6}{7} \div \frac{8}{11}$.

$= \frac{6}{7} \times \frac{11}{8}$ *Rewrite the division problem as a multiplication problem by finding the reciprocal of $\frac{8}{11}$, which is $\frac{11}{8}$.*

$= \frac{\overset{3}{\cancel{6}}}{7} \times \frac{11}{\underset{4}{\cancel{8}}}$ *Simplify by cross-canceling.*
The GCF of 6 and 8 is 2:
$6 \div 2 = 3$ and $8 \div 2 = 4$

$= \frac{33}{28} = 1\frac{5}{28}$

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EXAMPLE: Jay's landscaping has a few gas cans that can hold up to $5\frac{1}{4}$ gallons of gasoline to be used for their lawn mowers.

If the owner has a total of $12\frac{5}{6}$ gallons of gasoline, how many gas cans can he fill?

$$= 12\frac{5}{6} \div 5\frac{1}{4}$$

$$= \frac{77}{6} \div \frac{21}{4}$$

$$= \frac{77}{6} \times \frac{4}{21}$$

$$= \frac{\overset{11}{\cancel{77}}}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{21}}}$$

$$= \frac{22}{9} = 2\frac{4}{9}$$

Remember, in order to find the quotient you must first convert any mixed numbers to improper fractions.

The owner can fill $2\frac{4}{9}$ gas cans.

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CHECK YOUR KNOWLEDGE

Calculate the product or quotient.

1. $\frac{5}{8} \times \frac{14}{15}$

2. $\left(-1\frac{2}{7}\right) \times 1\frac{5}{9}$

3. $\left(-\frac{6}{11}\right) \times \left(-\frac{2}{3}\right)$

4. $\frac{2}{3} \div \frac{4}{5}$

5. $\left(-\frac{7}{4}\right) \div 4\frac{2}{3}$

6. $(-5) \div \left(-3\frac{1}{3}\right) \div \left(-\frac{1}{8}\right)$

7. $2\frac{4}{7} \times \frac{11}{12} \div \left(-1\frac{1}{21}\right)$

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Choose the correct method to find the answer.

8. $\frac{1}{3} \div \frac{6}{11}$

A. $\frac{1}{3} \times \frac{6}{11}$

B. $\frac{3}{1} \times \frac{11}{6}$

C. $\frac{1}{3} \div \frac{11}{6}$

D. $\frac{1}{3} \times \frac{11}{6}$

ANSWERS

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CHECK YOUR ANSWERS

1. $\frac{7}{12}$

2. -2

3. $\frac{4}{11}$

4. $\frac{5}{6}$

5. $-\frac{3}{8}$

6. -12

7. $-2\frac{1}{4}$

8. D

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Chapter 8

ADDING AND SUBTRACTING POSITIVE AND NEGATIVE FRACTIONS

ADDING POSITIVE AND NEGATIVE FRACTIONS WITH LIKE DENOMINATORS

To add fractions that have the same denominator, just add the numerators and keep the denominator.

EXAMPLE: Simplify $\frac{3}{5} + \frac{4}{5}$.

$= \frac{3+4}{5} = \frac{7}{5} = 1\frac{2}{5}$

EXAMPLE: Simplify $\left(-\frac{2}{11}\right) + \left(-\frac{8}{11}\right)$.

$\left(-\frac{2}{11}\right) + \left(-\frac{8}{11}\right)$ Both fractions are negative, so the answer is negative.

$= -\left(\frac{2}{11} + \frac{8}{11}\right)$

$= -\left(\frac{2+8}{11}\right) = -\frac{10}{11}$

EXAMPLE: Simplify $\left(-\frac{7}{9}\right) + \frac{2}{9}$.

$\left(-\frac{7}{9}\right)$ and $\frac{2}{9}$ have different signs, so subtract the absolute value of $\left(-\frac{7}{9}\right)$ and the absolute value of $\frac{2}{9}$:

$\left|-\frac{7}{9}\right| - \left|\frac{2}{9}\right| = \frac{7}{9} - \frac{2}{9} = \frac{5}{9}$

$-\frac{7}{9}$ has the greater absolute value, so the answer is negative:

$-\frac{5}{9}$

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SUBTRACTING POSITIVE AND NEGATIVE FRACTIONS WITH LIKE DENOMINATORS

To subtract negative fractions, rewrite the subtraction problem as an addition problem by using the additive inverse.

EXAMPLE: Simplify $\left(-\frac{5}{7}\right)-\left(-\frac{1}{7}\right)$.

$\left(-\frac{5}{7}\right)-\left(-\frac{1}{7}\right)$ — *Change into an addition problem. Use the additive inverse of $\frac{1}{7}$.*

$=\left(-\frac{5}{7}\right)+\left(\frac{1}{7}\right)$

$\left|-\frac{5}{7}\right|-\left|\frac{1}{7}\right|=\frac{5}{7}-\frac{1}{7}=\frac{4}{7}$ — Subtract the absolute values.

$-\frac{4}{7}$

This is the greater absolute value, so the answer is also negative.

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ADDING AND SUBTRACTING POSITIVE AND NEGATIVE FRACTIONS WITH UNLIKE DENOMINATORS

To add or subtract fractions with different denominators, we can create equivalent fractions that have the same denominators. We can do that by finding the **LEAST COMMON MULTIPLE (LCM)** of the denominators.

EXAMPLE Simplify $\frac{2}{5} + \frac{1}{4}$.

Step 1: Find the LCM of both denominators.

The multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, . . .
The multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, . . .

The **Least** Common Multiple of 5 and 4 is: 20.

Step 2: Rename the fractions as equivalent fractions.

Ask, 5 times what number equals 20? 4.

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Multiply the numerator and denominator by 4 to change to an equivalent fraction.

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

4 times what number equals 20? 5.

Multiply the numerator and denominator by 5 to change to an equivalent fraction.

$$\frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

Step 3: Add or subtract the fractions, and simplify.

$$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$

EXAMPLE: Simplify $\frac{1}{4} - \frac{5}{6}$.

Step 1: Find the LCM of both denominators.

The multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, . . .
The multiples of 6 are: 6, 12, 18, 24, 30, . . .

The Least Common Multiple of 4 and 6 is: 12.

Step 2: Rename the fractions as equivalent fractions.

$$\frac{1\times3}{4\times3} = \frac{3}{12} \quad \text{and} \quad \frac{5\times2}{6\times2} = \frac{10}{12}$$

Step 3: Subtract the fractions, and simplify.

$$= \frac{3}{12} - \frac{10}{12}$$

$$= \frac{3}{12} + \left(-\frac{10}{12}\right)$$

Change the subtraction into addition. $\frac{10}{12}$ *is the additive inverse of* $-\frac{10}{12}$

$$\left|-\frac{10}{12}\right| - \left|\frac{3}{12}\right| = \frac{10}{12} - \frac{3}{12} = \frac{7}{12}$$

Subtract the absolute values.

$-\frac{10}{12}$ has the greater absolute value, so the answer is also negative:

$$-\frac{7}{12}$$

CHECK YOUR KNOWLEDGE

Calculate. Simplify each answer if possible.

1. $\frac{7}{10} + \frac{5}{10}$

2. $\frac{5}{12} - \frac{7}{12}$

3. $-\frac{6}{7} + \left(-\frac{4}{7}\right)$

4. $-\frac{5}{8} + \frac{2}{3}$

5. $-3 - \left(-\frac{5}{6}\right)$

6. $4\frac{1}{8} - 2\frac{5}{8}$

7. $1\frac{3}{5} - \left(-7\frac{4}{5}\right)$

8. $2\frac{1}{4} - \left(-3\frac{1}{6}\right)$

9. $-14\frac{1}{2} + \left(-2\frac{4}{5}\right)$

10. May Ling has $9\frac{1}{4}$ chocolate bars. She gives Ahmad $2\frac{3}{5}$ of her chocolate bars. How many chocolate bars does May Ling have left?

ANSWERS

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CHECK YOUR ANSWERS

1. $1\frac{1}{5}$

2. $-\frac{1}{6}$

3. $-1\frac{3}{7}$

4. $\frac{1}{24}$

5. $-2\frac{1}{6}$

6. $1\frac{1}{2}$

7. $9\frac{2}{5}$

8. $5\frac{5}{12}$

9. $-17\frac{3}{10}$

10. May Ling has $6\frac{13}{20}$ chocolate bars left.

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Chapter 9

ADDING AND SUBTRACTING DECIMALS

To add or subtract decimal numbers, you can rewrite the problem vertically. First, line up the decimal points to align the place values of the digits. Next, add or subtract the same way you add or subtract whole numbers. Last, write the decimal point in the sum or difference.

EXAMPLE: Find the sum of 1.2 + 73.65.

$$\begin{array}{r} 1.2 \\ +\ 73.65 \\ \hline 74.85 \end{array}$$

Rewrite the problem vertically to align the place value of the digits.

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EXAMPLE: Find the sum of 56.09 + 7.8.

$$\begin{array}{r} 56.09 \\ +\ 7.80 \\ \hline 63.89 \end{array}$$

Rewrite the problem vertically to align the place value of the digits.

Think: 7.8 could be rewritten as 7.80.

Anytime you add a whole number and a decimal, include the decimal point to the right of the whole number.

EXAMPLE: Find the sum of 8 + 1.45.

Rewrite 8 as 8.00, so that there are the same number of digits after the decimal point as 1.45.

$$\begin{array}{r} 8.00 \\ +\ 1.45 \\ \hline 9.45 \end{array}$$

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ADDING DECIMALS WITH DIFFERENT SIGNS

To add decimal numbers with different signs, subtract the absolute value of the numbers. Then use the sign of the number with the greatest absolute value for the difference.

EXAMPLE: Find the sum of $-9.81 + 3.27$.

-9.81 and 3.27 have different signs. So, subtract their absolute values:

$$|-9.81| - |3.27| = 9.81 - 3.27$$

Rewrite the expression to align the place value of the digits.

$$\begin{array}{r} 9.81 \\ -\ 3.27 \\ \hline 6.54 \end{array}$$

-9.81 has the larger absolute value, so the answer is negative: -6.54

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SUBTRACTING DECIMALS WITH DIFFERENT SIGNS

Align the decimal points of each number and then subtract. Be sure to write the decimal point in the answer.

EXAMPLE: Calculate the difference of 8.01 − 5.4.

$$\begin{array}{r} 8.01 \\ -\ 5.40 \\ \hline 2.61 \end{array}$$

Rewrite the problem vertically to align the place value of the digits.

8.01 has the greater absolute value, so the answer is also positive: 2.61

EXAMPLE: Calculate the difference of −0.379 − 10.5.

$= -0.379 - 10.5$

$= -0.379 + (-10.5)$

Change the subtraction to an addition problem. -10.5 is the additive inverse of 10.5.

Add the absolute values of both numbers:

$|-0.379| + |-10.5| = 0.379 + 10.5$

Both numbers are negative, so the answer is also negative: -10.879

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EXAMPLE: A scientist boils a liquid to 142.07°F. The scientist then puts the liquid in a freezer where the temperature of the liquid decreases by 268.3 degrees. What is the final temperature of the liquid?

The temperature of the liquid decreases, so subtract:

142.07 - 268.3

Arrange vertically and align decimal points:

$$\begin{array}{r} 268.30 \\ \underline{-142.07} \\ 126.23 \end{array}$$

-268.3 has the greater absolute value. So, the answer is negative:

-126.23 degrees

The final temperature is -126.23°F.

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CHECK YOUR KNOWLEDGE

For questions 1 through 9, simplify each expression.

1. $9.6 + (-1.5)$

2. $7.1 + (-5.9)$

3. $-3.4 - 1.6$

4. $-7.3 - 3.9$

5. $3.1 - (-0.4)$

6. $0.15 - (-41.7)$

7. $-1.67 - (-5.9)$

8. $-5 + .07 + (-3.1)$

9. $-3.1 - (-8.67) + (-1.05)$

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10. Luis is asked to simplify the following expression: $-2.53 - (-1.26)$. His work has the following steps:

Step 1: $= -2.53 + (1.26)$

Step 2:
$$\begin{array}{r} 2.53 \\ +\ 1.26 \\ \hline 3.79 \end{array}$$

Step 3: -2.53 has the greater absolute value, so the answer is also negative: -3.79

However, Luis makes an error in his work.

On which step did Luis make an error? What should Luis have done?

ANSWERS

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CHECK YOUR ANSWERS

1. 8.1

2. 1.2

3. −5

4. −11.2

5. 3.5

6. 41.85

7. 4.23

8. −8.03

9. 4.52

10. Luis made an error in step 2. Because the numbers have different signs, Luis should have subtracted them, not added them.

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Chapter 10

MULTIPLYING AND DIVIDING DECIMALS

MULTIPLYING DECIMALS

To multiply decimal numbers, you don't need to line up the decimals.

Steps for multiplying decimals:

1. Count the negative signs to find the sign of the product.
2. Multiply the numbers the same way you multiply whole numbers. In other words, ignore the decimal points!
3. Place the decimal point in your answer: The number of decimal places in the answer is the total number of decimal places in the two original factors.

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EXAMPLE: Calculate the product of the following expression: 5.32×1.4

Step 1: Since there are no negative signs, the answer is positive.

Step 2: Multiply the numbers without the decimal point:

$$\begin{array}{r} 532 \\ \times \quad 14 \\ \hline 2128 \\ 5320 \\ \hline 7448 \end{array}$$

Step 3: Determine where the decimal point goes in the answer.

Since 5.32 has 2 digits to the right of the decimal point, and 1.4 has 1 digit to the right of the decimal point, the total number of decimal places is 3.

So the product is: 7.448.

EXAMPLE: Calculate the product of the following expression: $3.120 \times (-0.5)$.

Step 1: Since there is one negative sign, the answer is negative.

Step 2: Multiply the numbers without the decimal point:

$$\begin{array}{r} 3120 \\ \times \quad 5 \\ \hline 15600 \end{array}$$

Step 3: Determine where the decimal point goes in the answer.

The total number of decimal places is 4, so the product is -1.5600.

If there are zeros at the end, keep them while you multiply, but when you write the final answer remove the zeros:

-1.5600 has 4 decimal places, but can be written as -1.56.

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The **DIVIDEND** is the number that is being divided.
The **DIVISOR** is the number that "goes into" the dividend.
The answer to a division problem is called the **QUOTIENT**.

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

OR

$$\text{divisor}\overline{)\text{dividend}}^{\ \text{quotient}}$$

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EXAMPLE: Calculate the quotient of $2.8 \div 0.7$.

Step 1: Since there are no negative signs, the answer is positive.

Step 2: Multiply both the dividend, 2.8, and the divisor, 0.7, by 10, so that they both become whole numbers.

$2.8 \times 10 = 28$ and $0.7 \times 10 = 7$

$2.8 \div 0.7 = 28 \div 7$

Step 3: Divide the numbers: $28 \div 7 = 4$

EXAMPLE: Calculate the quotient of $(-6.912) \div 0.03$.

Step 1: Since there is one negative sign, the answer is negative.

Step 2: Multiply both the dividend and the divisor by 1,000, so that they both become whole numbers: 6,912 and 30.

Step 3: Divide

$= -(6912 \div 30)$

$= -230.4$

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EXAMPLE: Amina bikes 32.64 miles in 2.4 hours. If she keeps up the pace, how many miles does Amina travel each hour?

Step 1: Since there are no negative signs, the answer is positive.

Step 2: Multiply both the dividend and the divisor by 100, so that they both become whole numbers.

$32.64 \times 100 = 3264$ and $2.4 \times 100 = 240$

$32.64 \div 2.4 = 3264 \div 240$

Step 3: Divide

$3264 \div 240 = 13.6$

So, Amina travels on her bike 13.6 miles each hour.

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CHECK YOUR KNOWLEDGE

For questions 1 through 8, simplify each expression.

1. $7 \times (-3.2)$

2. -8.3×1.02

3. $(-0.3) \times (-1.07)$

4. $(-37.8) \div 9$

5. $(-235.6) \div 0.04$

6. $(-32.04) \div (-0.6)$

7. $(-0.0168) \div 0.00007$

8. $-1.2 \times 0.8 \div (-0.03)$

9. A machine pumps 2.1 gallons of water every 1.6 minutes. How many gallons does the machine pump each minute?

10. Sandy jogs 19.7 miles in 4.5 hours. How many miles does she jog each hour? Round your answer to the nearest hundredth.

ANSWERS

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CHECK YOUR ANSWERS

1. -22.4
2. -8.466
3. 0.321
4. -4.2
5. -5,890
6. 53.4
7. -240
8. 32
9. The machine pumps 1.3125 gallons each minute.
10. Sandy jogs 4.26 miles each hour.

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Unit 3

Ratios, Proportions, and Percents

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Chapter 11

RATIO

A **RATIO** is a comparison of two or more quantities. For example, you might use a ratio to compare the number of green jelly beans to the number of red jelly beans. A ratio can be written in various ways.

The ratio 5 green jelly beans to 4 red jelly beans can be written:

5 to 4 or 5:4 or $\frac{5}{4}$

When comparing group *a* to group *b* we write the ratio as:

a to *b* or *a*:*b* or $\frac{a}{b}$

We can let *a* represent the first quantity and *b* represent the second quantity.

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EXAMPLE: Thirteen students joined after-school clubs in September. Eight joined the drama club and five joined the chess club. What is the ratio of students who joined the drama club to students who joined the chess club?

8 to 5 or 8:5 or $\frac{8}{5}$

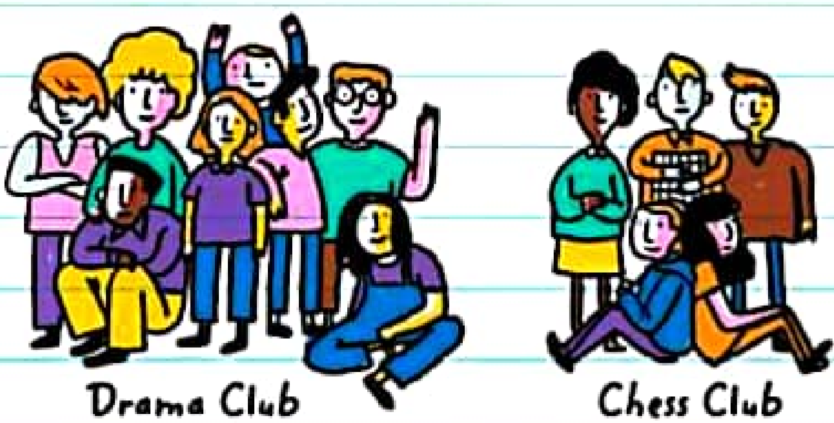
> Another way to say this is,
> "For every 5 students
> who joined the chess club,
> 8 students joined the drama club."

What is the ratio of students who joined the chess club to the total number of students who joined clubs?

5 to 13 or 5:13 or $\frac{5}{13}$

students who joined chess club

total number of students



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SIMPLIFYING RATIOS

We can simplify ratios just like we simplify fractions.

EXAMPLE: Janelle makes a beaded key ring. She uses 12 beads total. Among the 12 beads are 3 purple beads and 6 green beads. What is the ratio of purple beads to green beads? What is the ratio of green beads to the total number of beads?

The ratio of purple beads to green beads written as a fraction is $\frac{3}{6}$. This can be simplified to $\frac{1}{2}$.

So for every 1 purple bead, there are 2 green beads.

The ratio of green beads to the total number of beads used is $\frac{6}{12}$. This can be simplified to $\frac{1}{2}$.

So, 1 out of every 2 beads used is green.

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EQUIVALENT RATIOS

EQUIVALENT RATIOS have the same value. We can multiply or divide both *a* and *b* by any value (except zero), and the ratio *a* to *b* remains the same (equivalent).

For example, ratios that are equivalent to 3:5 include:

6:10	18:30	120:200
(3 × 2:5 × 2)	(3 × 6:5 × 6)	(3 × 40:5 × 40)

EXAMPLE: Find equivalent ratios for $\frac{18}{24}$.

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

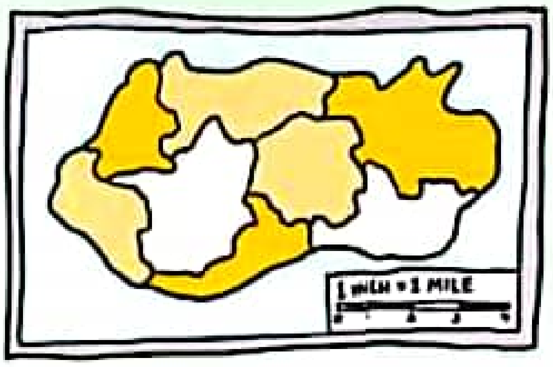
$$\frac{18}{24} = \frac{18 \div 3}{24 \div 3} = \frac{6}{8}$$

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

equivalent ratios

$\frac{18}{24}$ is equivalent to $\frac{9}{12}$, $\frac{6}{8}$, $\frac{3}{4}$, and many others.

A ratio is often used to make a scale drawing—a drawing that is similar to an actual object or place but bigger or smaller.



A map's key shows the ratio of the distance on the map to the actual distance in the real world.

CHECK YOUR KNOWLEDGE

For questions 1 through 5, write each ratio as a fraction. Simplify when possible.

1. 2:4

2. 3:5

3. 8 to 64

4. 5 to 30

5. For every 100 bottles of water, 25 were fruit flavored. Compare the number of fruit-flavored bottles of water to all bottles of water.

For questions 6 through 8, write a ratio in the form of a:b to describe each situation. Simplify when possible.

6. In a coding club there are 8 boys to every 10 girls.

7. The ratio of people who answered all the questions in a survey to the total number of people who took the survey is $\frac{35}{50}$.

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8. Mr. Jeffrey bought masks for the drama club's fundraiser. He bought 10 blue masks, 8 red masks, and 12 white masks. What was the ratio of white masks to total masks bought?

9. Write three ratios that are equivalent to 14:21.

10. Write three ratios that are equivalent to 1:5.

ANSWERS

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CHECK YOUR ANSWERS

1. $\frac{1}{2}$

2. $\frac{3}{5}$

3. $\frac{1}{8}$

4. $\frac{1}{6}$

5. $\frac{1}{4}$

6. 8:10; simplified: 4:5

7. 35:50; simplified: 7:10

8. 12:30; simplified: 2:5

9. Sample answers: 1:1.5, 2:3, 28:42

10. Sample answers: 2:10, 3:15, 4:20

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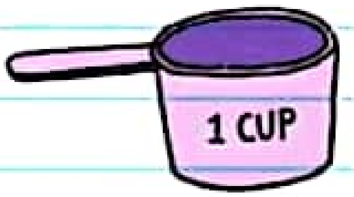
Chapter 12

UNIT RATE

A **RATE** is a special kind of ratio where the two amounts being compared have different units. For example you might use rate to compare 3 cups of water to 2 tablespoons of cornstarch. The units compared—cups and tablespoons—are different.

1 tablespoon



Rate: Units are different.

A **UNIT RATE** is a rate that has 1 as its denominator. To find a unit rate, set up a ratio as a fraction and then divide the numerator by the denominator.

> Unit rate compares an amount to one unit.

EXAMPLE: Jackson swims $\frac{1}{2}$ mile every $\frac{1}{3}$ hour. What is the unit rate of Jackson's swim?

This means, "How many miles per hour did Jackson swim?"

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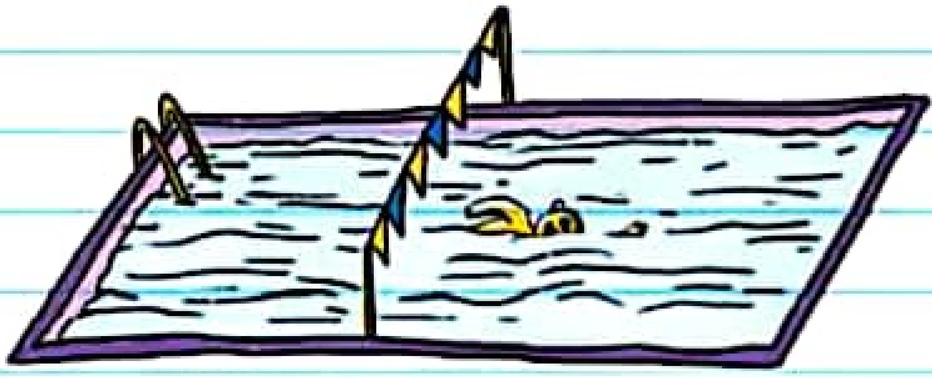
$$\frac{1}{2}\text{ mile} : \frac{1}{3}\text{ hour} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = \frac{1.5}{1}$$

$= 1\frac{1}{2}$ miles per hour

Jackson swims at a rate of $1\frac{1}{2}$ miles per hour.



EXAMPLE: A car can travel 300 miles on 15 gallons of gasoline. What is the unit rate per gallon of gasoline?

divide

$$300\text{ miles} : 15\text{ gallons} = \frac{\cancel{300}\text{ miles}}{\cancel{15}\text{ gallons}} = \frac{20}{1} = 20\text{ miles per gallon}$$

The unit rate is 20 miles per gallon.

This means that the car can travel 20 miles on 1 gallon of gasoline.

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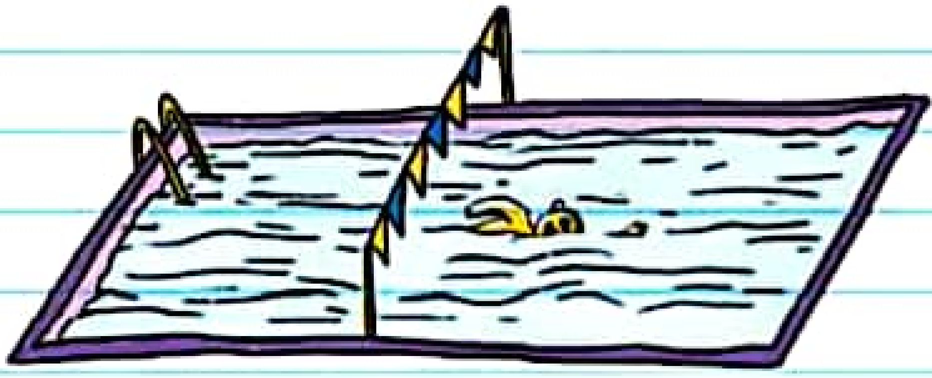

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